

On Pre-Fascicle 8A, Exercise 11

A closely related published result, and a small correction to the answer

Shisheng Li
daizisheng@gmail.com

April 19, 2026

1 Purpose of this note

In Pre-Fascicle 8A (§7.2.2.4, F8A: 10 Apr 2026 @ 1737), Exercise 11 asks for which vertices v the generalized Petersen graph $GP(2q, 2)$ admits a Hamiltonian path from $0'$ to v . While reading the answer, I noticed two things I would like to bring to your attention:

1. There is a 2013 paper by R. B. Richter that states, apparently independently, the same characterization of Hamilton paths in $P(n, 2)$ as a consequence of a general theorem covering all residue classes $n \pmod 6$. Specialising to $n = 2q$ and $x = 0'$ matches your B_q . Because your material has been under preparation for many years, I cannot tell whether your derivation predates Richter's publication; I merely record the connection for your reference. Richter's paper itself does not cite TAOCP.
2. Inside your answer, the exception clause "Unless $q < 4$: $B_3 = \{b_1, b_2\}$ " is in tension with the rest of the formula. The main formula, applied at $q = 3$, already gives $B_3 = \emptyset$, and I exhibit below a Hamilton path from a_0 to b_1 in $GP(6, 2)$ which shows that $b_1 \notin B_3$. The exception clause therefore seems to need correction (most simply, deletion).

These two items are independent. Item 2 stands on its own as a small erratum, regardless of attribution. Item 1 is for information only.

2 A closely related published result

The following paper gives a complete description of which pairs of vertices in $P(n, 2)$ are joined by a Hamilton path:

R. B. Richter, "Hamilton paths in generalized Petersen graphs," *Discrete Mathematics* **313** (2013), 1338–1341, Theorem 2.2.
DOI: 10.1016/j.disc.2013.02.021

with the immediate precursor

B. Alspach and J. Liu, "On the Hamilton connectivity of generalized Petersen graphs," *Discrete Math.* **309** (2009), 5461–5473.
DOI: 10.1016/j.disc.2008.12.016

Richter's Theorem 2.2 is stated for every residue class $n \pmod 6$ and covers all pairs of endpoints, not

only pairs starting at $0'$. Specialising to $n = 2q$ and $x = 0'$ gives a characterisation that matches your B_q set in each of the three residue classes of $q \bmod 3$.

Two caveats worth flagging:

- Richter’s bibliography does not cite *The Art of Computer Programming*. The paper’s references are only Alspach–Liu (2009), Alspach–Qin (2001), Chen–Quimpo (1981), and Holton–Sheehan (1993). So Richter appears not to have been aware of your material on this topic.
- Pre-Fascicle 8A has been under preparation over a long period, and the B_q characterisation may well be an independent earlier finding of yours that simply had not appeared in print. I do not know the history, and I make no claim about priority.

Notation dictionary. Knuth’s outer-ring vertices are $0, 1, \dots, n - 1$ and inner-ring vertices are $0', 1', \dots, (n - 1)'$, with the $k = 2$ specialisation $a_j = (2j)'$, $b_j = 2j$, $c_j = 2j + 1$, $d_j = (2j + 1)'$. Richter writes outer-ring vertices as u_i and inner-ring vertices as v_i . So v_i in Richter corresponds to i' in Knuth, and u_i to i .

3 The exception clause, and why it looks suspicious

The published answer concludes:

It turns out that Hamiltonian paths from a_0 to v exist except when v lies in B_q , where

$$B_q = \{a_j \mid j \bmod 3 = 0, 0 < j < q\} \text{ when } q \bmod 3 = 0;$$

$$B_q = \{a_j \mid j \bmod 3 = 2, 0 < j < q\} \text{ when } q \bmod 3 = 1;$$

$$B_q = \{a_j \mid j \bmod 3 \neq 1, 0 < j < q\} \cup \{c_0, c_{q-1}\} \cup \{b_j \mid j \bmod 3 = 1, 0 < j < q\} \\ \text{when } q \bmod 3 = 2.$$

(Unless $q < 4$: $B_3 = \{b_1, b_2\}$.)

At $q = 3$ the main formula sits in the $q \bmod 3 = 0$ branch, where j ranges over $\{1, 2\}$, neither of which is $\equiv 0 \pmod{3}$; so the main formula already gives $B_3 = \emptyset$. The exception clause overrides this to $B_3 = \{b_1, b_2\}$ —a substantive assertion that $a_0 \rightarrow b_1$ and $a_0 \rightarrow b_2$ Hamilton paths do not exist in $\text{GP}(6, 2)$. A concrete construction shows this assertion is incorrect.

4 An explicit Hamilton path $a_0 \rightarrow b_1$ in $\text{GP}(6, 2)$

Consider the 12-vertex walk in $\text{GP}(6, 2)$

$$a_0 \rightarrow a_1 \rightarrow a_2 \rightarrow b_2 \rightarrow c_1 \rightarrow d_1 \rightarrow d_0 \rightarrow d_2 \rightarrow c_2 \rightarrow b_0 \rightarrow c_0 \rightarrow b_1.$$

All twelve vertices are distinct, and every consecutive pair is an edge of $\text{GP}(6, 2)$:

Step	Edge (Knuth)	Edge (primed)	Type
1	$a_0 - a_1$	$0' - 2'$	inner $i' - (i+2)'$
2	$a_1 - a_2$	$2' - 4'$	inner
3	$a_2 - b_2$	$4' - 4$	spoke
4	$b_2 - c_1$	$4 - 3$	outer 6-cycle
5	$c_1 - d_1$	$3 - 3'$	spoke
6	$d_1 - d_0$	$3' - 1'$	inner
7	$d_0 - d_2$	$1' - 5'$	inner
8	$d_2 - c_2$	$5' - 5$	spoke
9	$c_2 - b_0$	$5 - 0$	outer (mod 6)
10	$b_0 - c_0$	$0 - 1$	outer
11	$c_0 - b_1$	$1 - 2$	outer

So $b_1 \notin B_3$. By the automorphism of $GP(6, 2)$ that exchanges the two inner triangles $\{a_0, a_1, a_2\} = \{0', 2', 4'\}$ and $\{d_0, d_1, d_2\} = \{1', 3', 5'\}$ (together with the corresponding outer-ring swap), the reflected walk also shows $b_2 \notin B_3$. A picture of the path is given in Figure 1.

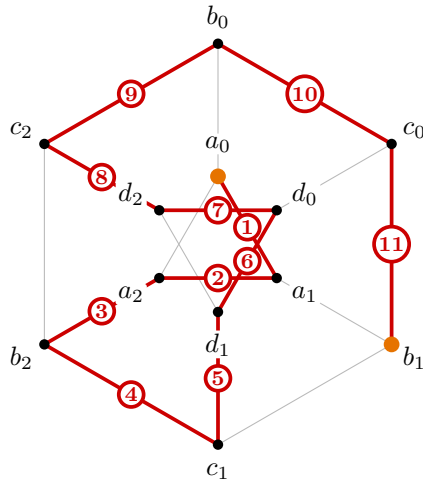
5 Suggested correction

The simplest fix is to delete the exception clause:

“(Unless $q < 4$: $B_3 = \{b_1, b_2\}$.)” ← remove this line.

With the exception removed, the main formula evaluated at $q = 3$ gives $B_3 = \emptyset$, which the construction above confirms: every vertex of $GP(6, 2)$ is reachable from a_0 by a Hamilton path.

Prepared with the assistance of Claude Code (Anthropic); the Hamilton path above was verified by hand against the edge list of $GP(6, 2)$.



Start: a_0 (orange) End: b_1 (orange) Red numbers 1–11 are the Hamilton path edges in order.

Figure 1: $GP(6, 2)$ in concentric-ring form. Gray lines are the full edge set. The red path (edges numbered 1–11) is the Hamilton path $a_0 \rightarrow b_1$ given in the table above. Orange dots mark the endpoints a_0 (start) and b_1 (end).